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Spacetime quotients, Penrose limits and restoration of conformal symmetry

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Abstract: In this paper we study the Penrose limit of AdS_5 orbifolds. The orbifold can be either in the pure spatial directions or space and time directions. For the $AdS_5/\Gamma \times S^5$ spatial orbifold we observe that after the Penrose limit we obtain the same result as the Penrose limit of $AdS_5 \times S^5/\Gamma$. We identify the corresponding BMN operators in terms of operators of the gauge theory on $R \times S^3/\Gamma$. The semi-classical description of rotating strings in these backgrounds have also been studied. For the spatial AdS orbifold we show that in the quadratic order the obtained action for the fluctuations is the same as that in S^5 orbifold, however, the higher loop correction can distinguish between two cases.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence, D-branes.

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Contents

1. Introduction

In the past five years important results have been obtained relating closed and open string theories, in particular the AdS/CFT correspondence [1] had important impact into the understanding between these two sectors. The main problem in testing the conjecture beyond the supergravity level is the fact that we do not know how to quantize string theory in the presence of RR-fluxes. More recently, the authors of [2] have considered the sector of string states with large angular momentum along the central circle of S^5 (obtained in [3]) and have compared the quantum string oscillations with the spectrum of anomalous dimensions of field theory operators with large R-charge. This is the Penrose limit of the $AdS_5 \times S^5$ which was also discussed in [4] yielding the maximal supersymmetric pp-wave. The results of [2] have been generalized to sphere orbifolds and other lower-supersymmetric backgrounds [5]–[8] and the maps between string oscillators and definite operators of the field theory have been identified.

Another approach has been initiated in [9] and expanded in [10] (see also [11, 12]). It consists of choosing a classical solution of the string action and making an expansion in R^2/α' which are mapped into field theory energy spectrum and its 1-loop quantum corrections.

JHEP01(2003)028

In the current project, we consider the issue of AdS orbifolds for which the above map has not yet been identified. In this case, the field theories do not live on $R \times S^3$ but on $R \times S^3/\Gamma$ and the usual orbifold theory approach does not apply. The way to deal with these theories was outlined in [13] for field theories living on general M/Γ manifolds where the orbifold action on M is free and hence there is no singular locus. It is argued in [13] that the field theories should be modified by introducing a non-trivial flat connection which relates fields living on different patches of the manifold M.

The main observation is that by taking the Penrose limit, the field corresponding to the boosted direction is required to live only on one patch and the covariant derivative is taken only with respect to the gauge field which corresponds to the same patch. More precisely, fields living in different patches lead to the twisted sectors for strings on plane wave orbifolds. The fields which are orthogonal to the boosting direction are unchanged. In this limit, the theory maps naturally into the Penrose limit of the $\mathcal{N}=2, S^5/\Gamma$ orbifolds which were discussed in [6, 7], and the different patches are mapped into the multiple coverings of S^5/Γ.

This map should also appear in the "semi-classical" approach of ref. [10]. Performing the calculations we show that indeed the large angular momentum sector of strings on $AdS_5 \times S^5/\Gamma$ and $AdS_5/\Gamma \times S^5$ are the same for the quadratic corrections. In fact the conclusion is that the quadratic corrections are identical, but the quartic corrections appear with a different sign.

In section 2 we discuss the two types of orbifolds of AdS_5, one corresponding to a space orbifold and the other to a space-time orbifold (the complete discussion has appeared in [14]). We will concentrate in section 3 on the field theory dual to a space AdS_5 orbifold and leave the space-time orbifold for a future work. In section 4 we discuss the Penrose limits of the two types of AdS_5 orbifolds and the BMN operators for our case. In section 5 we apply the method of [9, 10] to our cases and compare the quantum corrections.

2. Half supersymmetric orbifolds of AdS_5

In this section we discuss quotients of AdS spaces (see [14, 15, 16] for previous discussions). The $AdS_5 \times S^5$ space can be embedded in a twelve dimensional space with two time coordinates, i.e. the AdS_5 piece is sitting in a (4+2)-dimensional space and S^5 in a six dimensional euclidean space.

Let us first consider the AdS_5 part and let z, u and v be three complex coordinate satisfying

$$-|z|^2 + |u|^2 + |v|^2 = -\hat{R}^2\,, \tag{2.1}$$

where $\hat{R}$ is the radius of the AdS_5.

In general the AdS_5 space has a SO(4, 2) global symmetry and one can orbifold the space by a (discrete) subgroup Γ of that. Then as discussed in [14] half supersymmetric orbifolds of AdS_5 can be obtained in two different ways

1. Space orbifold

$$u \equiv \gamma u\,, \qquad v \equiv \gamma^{-1} v\,,$$

2. Space-time orbifold

$$z \equiv \gamma z\,, \qquad v \equiv \gamma^{-1} v\,,$$

where $\gamma \in \Gamma$ and for the case of $\mathbf{Z_k}$ orbifold $\gamma = e^{2\pi i/k}$.

The case one corresponds to modding out by a subgroup of SO(4) $\in$ SO(4, 2) and hence the symmetry of the quotient space is SU(2) $\times$ U(1). This orbifold has a fixed line (the global time). The second one is obtained by quotienting with SO(2, 2) $\in$ SO(4, 2) and has a fixed circle and the final surviving symmetry is SU(1, 1) $\times$ U(1).

The metric for AdS/Γ orbifolds is of course similar to the simple AdS_5 metric, where some of the angular variables have been limited to range from zero to $2\pi/k$. However, for our purpose (taking the Penrose limit) it would be more convenient to choose the global coordinate system. Let us first focus on the first case, for which it is more convenient to use the following parameterization for the complex coordinates

$$\begin{aligned} z &= \hat{R}\cosh\rho\; e^{it} \\ u &= \hat{R}\sinh\rho\;\cos\delta\; e^{i\frac{\phi}{k}} \\ v &= \hat{R}\sinh\rho\;\sin\delta\; e^{i(\frac{\phi}{k}+\gamma)}\,, \end{aligned} \tag{2.2}$$

where all the angular coordinates are now ranging from zero to 2π and $\rho \in [0,\infty)$. The metric in the above coordinates takes the form

$$\begin{aligned} ds^2 &= -dzd\bar{z} + dud\bar{u} + dvd\bar{v} + ds^2_{S^5} \\ &= \alpha' R^2\Bigg[-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\left(d\delta^2 + \frac{1}{k^2}\cos^2\delta d\phi^2 + \sin^2\delta\left(\frac{1}{k}d\phi + d\gamma\right)^2\right) + \\ &\qquad + \cos^2\alpha\, d\theta^2 + d\alpha^2 + \sin^2\alpha\, d\Omega_3^2\Bigg]\,. \end{aligned} \tag{2.3}$$

Here we have used a notation in which $R^2 = \hat{R}^2/\alpha'$ is dimensionless and ρ is a radial coordinate transverse to time. The boundary of the spacetime, which is at $\rho = \infty$, is $R \times S^3/\Gamma$ and this is where the dual gauge theory resides. We note that although the bulk AdS_5/Γ has a fixed circle (which can be extended to a real line, the global time), for the boundary Γ is freely acting on S^3 and hence there is no singular locus there. Due to orbifolding, the dual gauge theory which is defined on $R \times S^3/Z_k$ is not Lorentz invariant.[1] We would also like to recall that the AdS orbifold preserves 16 supercharges, and hence the dual gauge theory is a $\mathcal{N} = 2,\ D = 4$ theory.

For orbifolds of the second type, however, it is better to use the parameterization:

$$\begin{aligned} z &= \hat{R}\cosh\rho\; e^{i\frac{t}{k}} \\ u &= \hat{R}\sinh\rho\;\cos\delta\; e^{i(\frac{t}{k}+\phi)} \\ v &= \hat{R}\sinh\rho\;\sin\delta\; e^{i\gamma}\,, \end{aligned} \tag{2.4}$$

[1] To see this we note that after a Wick rotation, one can conformally map $R \times S^3/Z_k$ to R^4/Z_k. Then it is manifestly seen that the orbifolding breaks the SO(4) (euclidean) Lorentz symmetry to SU(2).

and hence the metric reads as

$$ds^2 = \alpha' R^2 \Big[-\frac{1}{k^2}\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho \Big(d\delta^2 + \sin^2\delta d\gamma^2 + \cos^2\delta \left(\frac{1}{k}dt + d\phi\right)^2 \Big) + + \cos^2\alpha\, d\theta^2 + d\alpha^2 + \sin^2\alpha\, d\Omega_3^2 \Big], \tag{2.5}$$

where $0 \leq t,\ \phi \leq 2\pi$. The boundary of this orbifold,where the dual gauge theory resides, is actually $S^1/\Gamma \times S^3$. In this case we have closed time-like curves because of the orbifold fixed points on the boundary. Therefore, unlike the usual AdS case, it is not possible to extend over the circle.

Similarly one can also consider orbifolds of S^5 [17, 18]. In this case, however, there is only one choice for the half supersymmetric orbifold where the quotient space has a fixed circle with the SU(2) × U(1) symmetry remaining (out of the global SO(6) rotation symmetry). Therefore we have three kinds of half supersymmetric orbifolds of $AdS_5 \times S^5$. A similar classification can also be used for the (half SUSY) $AdS_{4,7} \times S^{7,4}$ orbifolds.

3. Field theories duals to $AdS_5 \times S^5$ orbifolds

In this section we will discuss the field theories which are dual to the AdS_5 orbifolds discussed in the previous section. We will also compare the results to the case of S^5 orbifold treated extensively in the literature [17, 18]. For simplicity, we only consider the case of A_n orbifolds i.e. $\Gamma = \mathbf{Z_n}$.

For $\mathbf{Z_k}$ orbifolds of S^5 which leave a fixed S^1, one starts in the covering space with a gauge group $\mathrm{U}(kN)$ and then projects out the $\mathbf{Z_k}$ invariant components. If the S^5 is described by three complex scalar fields $\phi_i, i = 1, 2, 3$, then the projection is ensured by the condition:

$$\Omega\, Z(x)\, \Omega^{-1} = Z(x)\,, \tag{3.1}$$

$$\Omega\, \phi_j(x)\, \Omega^{-1} = \omega \phi_j(x) \tag{3.2}$$

where we can choose $Z(x) = \phi_3(x)$ to denote the fixed S^1, $\phi_j(x)$ the other two complex coordinates and $\Omega = \mathrm{diag}(1, \omega, \ldots, \omega^{k-1})$. The projection Γ is global (is the same for any point x). There is also an action on the gauge fields A_μ which implies that they can also be diagonalized in k $N \times N$ blocks. The resulting spectrum is that of a four dimensional $\mathcal{N} = 2$ quiver theory with the gauge group $(\mathrm{U}(N))^k$ and bifundamentals. The fields A_μ, Z together with the spinors form k, $\mathcal{N} = 2$ vector multiplets and the bifundamentals form hypermultiplets. The beta function vanishes for each of the gauge groups $\mathrm{U}(N)$ inside $(\mathrm{U}(N))^k$. In the supergravity side this corresponds to the existence of a fixed line corresponding to the dilaton for solutions of type $AdS_5 \times S^5/Z_k$ as stated in [17]. There are also twisted sectors which live on the fixed locus of the Z_k action i.e. $AdS_5 \times S^1$, and their spectrum has been compared to the spectrum of chiral operators in field theory [19].

Now let us consider the field theory dual to the AdS_5 orbifolds discussed in the previous section. To do that, we will review arguments of [13]. Their results, however, is more general than S^3/Γ orbifold and is applicable to gauge theories on M/Γ manifolds where M

JHEP01(2003)028

is any compact Riemann manifold and Γ is any freely acting discrete isometry group. If we restrict ourselves to the case $\Gamma = \mathbf{Z_k}$, a field ϕ on $M/\mathbf{Z_k}$ transforming under the adjoint action of a gauge group $\mathrm{U}(kN)$ can be characterized as a pull-back $s^*\tilde{\phi}$, by a local section $s: M/\mathbf{Z_k} \to \mathbf{M}$, of a map $\tilde{\phi}: M \to C^{kN} \otimes C^{kN}$ transforming as

$$\tilde{\phi}(u \cdot \gamma) = \mathcal{R}(\gamma^{-1})\tilde{\phi}(u)\mathcal{R}(\gamma) \tag{3.3}$$

where u belongs to M and γ to $\mathbf{Z_k}$ and $\mathcal{R}(\gamma)$ is a representation of $\mathbf{Z_k}$ and an element of $\mathrm{U}(k) \times \mathbf{1}_{N\times N}$.

We can now state the difference between the S^5 and the AdS_5 orbifolds. Note that while the conditions (3.1), (3.2) constrain the entries of the matrices $Z(x)$ (the r.h.s. and l.h.s. being at the same point), the condition (3.3) relate fields at different points u and $u\cdot\gamma$. In other words, the orbifolding is defined by the "twisted boundary condition" for the ϕ field on $M/\mathbf{Z_k}$ as given in (3.3). One may remove $\mathcal{R}(\gamma)$ elements by performing a gauge transformation. However, this gauge transformation is not globally defined, i.e. due to the non-trivial holomony of the gauge field A the gauge transformation is not single valued when we go from one patch to another. Therefore we are forced to introduce a non-trivial flat connection [13]. In short, the fields ϕ on $M/\mathbf{Z_k}$ in the presence of non-trivial flat connection are the equivariant (as defined in (3.3)) subclass of all the field defined on M. This is to be compared with the case of the S^5 orbifold, where the constraints on the fields appear only when we go to the covering space and those constraints are on the gauge group indices, not on the x coordinate.

In order to describe the field theory dual to AdS_5 orbifolds, we start with a theory on $R \times S^3$ i.e $\mathcal{N} = 4$, $\mathrm{U}(kN)$ theory with 3 scalar fields in the adjoint representation $\Phi_i, i = 1,2,3$ together with a gauge field A_μ. Let us focus on the modifications after modding out by $\mathbf{Z_k}$, necessary to obtain a theory on $R \times S^3/\mathbf{Z_k}$. In the supergravity side, the number of modes per unit volume does not change and the holographic principle tells us that the number of modes per unit volume in the field theory should be the same too. In fact in [13] it is demonstrated that there exists a flat connection A such that for every value of the laplacian on S^3 (with zero connection) there exists solutions to $D_A^2\psi = \lambda\psi$ on $S^3/\mathbf{Z_k}$ and the number of such modes is $1/k$ times the number of corresponding modes in S^3. Since the volume of S^3 is also reduced by $1/k$ after the projection, the number of modes per unit volume remains the same.

The action of $\mathbf{Z_k}$ breaks the Lorentz group in the field theory side and the theory is not conformal anymore. This is not an obvious statement and we now explain it. The conformality breaking can be traced back to either a non-vanishing beta function or to some scale introduced in the theory. We use the latter to explain non-conformality of our theory. The argument is similar to that of [20] where $\mathrm{U}(N)$ field theory was obtained on one brane wrapped N times on a circle of radius L. For the wrapped brane there are only N types of open strings on a circle of length NL, which has lower energy states and corresponds to adding the flat connection in the gauge theory. The flat connection is proportional to $1/L$ so introduces a scale in field theory. The same appears in our case, for the $\mathrm{U}(N)$ field theory on $S^3/\mathbf{Z_k}$ where the flat connection will depend on $1/\mathrm{vol}(S^3)$ which becomes a scale in the field theory and this makes the field theory non-conformal. Another explanation for the non-conformality would be the non-zero value of the beta function. This is implied by the fact that when calculating the beta function, one has to use fields belonging to different patches, which introduce summation over extra phases in the beta function and those summations do not add up to zero (it is proportional to $\sin^2(2\pi/k)$.)

4. Penrose limits and dual field theories

Now we are ready to zoom-in onto a light-like geodesic and take the Penrose limit. In general we can choose this geodesic inside AdS_5 or inside S^5. For the geodesics inside AdS_5 we may follow those which are parallel to the boundary (i.e. we boost along a circle inside S^3 part of AdS_5 space). It is not hard to see that these paths inside the AdS_5 only become exact light-like geodesics when we are at strict $\rho = \infty$ which is the boundary. One can show that the Penrose machinery does not work for such geodesics which are not formally in the space-time. More explicitly, if we take the $R \to \infty$ limit in this case and scale the other AdS_5 coordinates properly to keep metric components finite we end up with a metric which is not a solution of supergravity equations. Hence we only focus on the geodesics which are in the S^5 part.

For the three possible half supersymmetric orbifolds mentioned above, one can take Penrose limits in four different ways: two corresponding to the AdS_5 orbifolds and two corresponding to S^5 orbifolds.[2] The two sphere orbifold cases have been discussed in [6, 7, 8], where it is shown that, if we choose our geodesic along the fixed circle of the orbifold, we get a half supersymmetric plane wave (which can be thought of as the orbifold of the maximally supersymmetric plane wave). If we boost along the circle where the orbifolding is acting, we will get maximally supersymmetric orbifold (in this case the Penrose limit washes out the orbifolding information). As for the two other possibilities, coming from AdS_5 orbifolds, the case of space-time orbifold (we have called this case one) has been briefly discussed in [8] and shown that Penrose limit of this case leads to again an orbifold of maximally supersymmetric plane wave. However, the Penrose limit of the case two has not been explored yet.

4.1 Field theory dual to the Penrose limit of the spatial AdS_5 orbifold

We start by reviewing the Penrose limit of the space AdS_5 orbifold [8]. Consider the metric (2.3) and perform the following Penrose limit

$$x^+ = \frac{1}{2}(t+\theta)\,, \qquad x^- = \frac{1}{2}R^2(t-\theta)\,,$$

[2] We would like to discuss a possible subtlety which may arise in applying the Penrose method [23] to orbifold space-times. By construction Penrose method is a procedure defined in classical general relativity (GR). However orbifold spaces, due to singularities, are not generally well-defined objects in classical GR. In the half supersymmetric AdS space orbifold or S^5 orbifold the singular locus is $R \times S^5$ or $AdS_5 \times S^1$, respectively. Then, one may focus on a light-like geodesic which completely lies in the singular locus of the space (e.g. $\rho = 0,\ \alpha = 0,\ \theta = t$ of the metric (2.3)). For such geodesics, the scaling are so that we do not touch the orbifold structure. In such cases orbifolding and taking the Penrose limit commute. These are the cases in which Penrose limit does not enhance supersymmetry. We would like to thank M. Rangamani for a discussion on this point.

$$\rho = \frac{r}{R}\,, \qquad \alpha = \frac{x}{R}\,, \qquad R \to \infty \tag{4.1}$$

and x, r, x^+ and x^- fixed. Taking the limit we find

$$ds^2 = -4dx^+dx^- - (r^2 + x^2)(dx^+)^2 + dr^2 + dx^2 + x^2 d\Omega_3^2 + r^2 d\Omega_3'^2\,, \tag{4.2}$$

where the $d\Omega_3'^2$ is the metric for S^3/Z_k.

As we see from the previous formulas, after taking the Penrose limit, one cannot distinguish whether the orbifolding was originally in the AdS or sphere parts. At the level of supergravity or string theory, this is expected as we boost along the same direction in both cases for which we have an S^3 orbifold. This may be understood as follows: after taking the Penrose limit on $AdS_5 \times S^5$ we find a solution with SO(4) $\times$ SO(4) symmetry and the two SO(4)'s are at the same footing, i.e. there is a Z_2 symmetry which exchanges them [24]. From the AdS point of view one of the SO(4)'s is coming as a subgroup of SO(4,2), the AdS_5 isometry group, and the other as a subgroup of SO(6) symmetry of S^5. However, this difference is washed out by the Penrose limit. Now one may orbifold by a $\mathbf{Z_k}$ subgroup of either of the SO(4)'s. However from the field theory point of view, this tells us that a subsector of a conformal theory (the one for S^5 orbifold) is identical to a subsector of a non-conformal theory (the one for AdS_5 orbifold).

Our task here is to work out the explicit form of BMN operators in terms of the gauge theory on $R \times S^3/\mathbf{Z_k}$. Similar operators have been constructed in terms of quiver gauge theory operators [6, 7]. First we need to recognize what are the correspondents of the string light-cone momentum and energy. Although the full conformal group for the gauge theory on $R \times S^3/\mathbf{Z_k}$ is broken, we still have U(1) $\times$ (SU(2) $\times$ U(1)) subgroup of SO(4,2), where the eigenvalues for the first U(1) factor are the conformal dimension of the operators, Δ. Besides that, we also have the full SO(6) R-symmetry group. Following BMN, let us pick states carrying charge J under a U(1) factor of the R-symmetry. The Penrose limit (4.1) in the gauge theory side corresponds to $\Delta, J \to \infty$ while keeping

$$\begin{aligned} 2p^- &= i\partial_{x^+} = i(\partial_t + \partial_\theta) = \Delta - J \\ 2p^+ &= i\partial_{x^-} = \frac{i(\partial_t - \partial_\theta)}{R^2} = \frac{\Delta + J}{R^2}\,, \end{aligned}$$

fixed.

As the first step, we should identify the operators corresponding to vacuum for strings on plane wave orbifolds. In particular we note that we should have $k-1$ twisted and one untwisted vacuum states. To start with let us recall that gauge theories on $R \times S^3$ (after a Wick rotation) are related to theories on R^4 (or more precisely $R^4 - \{O\}$) through radial quantization, where translation along the original time direction corresponds to dilatation (scaling) for the theory on R^4 and $t = -\infty$ to the origin of R^4, O.[3] Then it is clear that gauge theory on $R \times S^3/\mathbf{Z_k}$ is obtained through the radial quantization of the same gauge theory on $R^4/\mathbf{Z_k}$. Next, we recall that the BMN operator corresponding to vacuum is

$$|\mathrm{vac}\rangle \longleftrightarrow \mathrm{Tr}\, Z^J(x)|0\rangle|_{x\to 0}$$

[3] Note that the R^4 metric $dr^2 + r^2 d\Omega_3^2$ is conformal to $dr^2/r^2 + d\Omega_3^2$.

where x is a point on R^4 and Z is the complex scalar field whose phase corresponds to the direction along which we have boosted in the Penrose limit. For the orbifold case, however, taking the $x \to 0$ limit is not trivial and depending on which patch of R^4 we start with, we will find k different answers, corresponding to the k vacua we were looking for. These k vacua are then related by orbifold action defined earlier in section 3. Explicitly, using (3.3) these different vacua are

$$|\mathrm{vac}\rangle_q \longleftrightarrow \mathrm{Tr}[S^q Z^J]\,, \qquad S = \mathrm{diag}\left(1, e^{2\pi i/k}, \ldots, e^{2(k-1)\pi i/k}\right) \tag{4.3}$$

where now Z has been defined only for one patch. It is useful to compare the above to the case of S^5 orbifolds. In that case, having twisted vacua stems from the fact that the gauge group is $(\mathrm{U}(N))^k$ and hence there are k $Z(x)$ fields (in the adjoint representations of each gauge factor). In terms of $kN \times kN$ matrices, it has exactly the same form as (4.3), but the "twisting phase" comes from a different origin [6]. For the AdS orbifold case, this comes from the fact that all the fields, including Z, on different patches are related by a non-trivial gauge transformations, while for the S^5 orbifold, that is just manifestation of having a gauge theory with k copies of $\mathrm{U}(N)$ factor with the certain quiver matter content.[4]

Given the vacuum states one may proceed with insertions of the covariant derivative on $R^4/\mathbf{Z_k}$, D^q_a, $a = 1,2,3,4$ and $q = 1,\ldots,k$ and other four scalars $\phi_{a'}$ into the string of Z's with proper phases. Since the situation would be quite similar to that of S^5 orbifolds we do not repeat them here. (They can be found in refs. [6, 7].) As an example, let us discuss the first level operators. The field Z is in the adjoint representation of the gauge group so the same discussion applies for the vector field. Therefore the quantities $D_a Z = \partial_a Z + [A_a, Z]$ (where D_a is the covariant derivative for the gauge theory on R^4 and A_a is the gauge field) will also be defined only for one patch and it will give k different answers for the k vacua. Therefore we can define the quantities:

$$\mathrm{Tr}[S^q\ D_a Z]\,, \qquad S = \mathrm{diag}\left(1, e^{2\pi i/k}, \ldots, e^{2(k-1)\pi i/k}\right) \tag{4.4}$$

as the twist invariant operators with $\Delta - J = 1$.

With this construction it is clear that the BMN sector of the dual gauge theories for AdS_5 and S^5 orbifold cases should be equivalent and both should describe the strings on the half supersymmetric plane wave orbifolds.

It is also important to note that since the plane wave background obtained from taking the Penrose limit of AdS is the same as the one obtained from the S^5 orbifold and, moreover, since the latter one is dual to a subsector of a conformal theory, we might conclude that the conformal symmetry is restored for the BMN subsector of the gauge theory on $R \times S^3/\mathbf{Z_k}$. This situation is very similar to the supersymmetry enhancement in the Penrose limit procedure [5, 6, 7].

[4]This "similarity" can also be understood in the context of the "holographic" model proposed in [26], where the boundary of the plane wave is one dimensional as a matrix model being built out of the lowest lying KK modes of the SYM on an S^3 compactification. The discussion of [26] can be used for $S^3/\mathbf{Z_k}$ as well. Then, in this point of view it is evident that in the $S^3/\mathbf{Z_k}$ orbifold case we should have k different vacua.

JHEP01(2003)028

As a straightforward generalization of the above discussions one may consider the orbifolds $AdS_5/\Gamma \times S^5/\Gamma'$, where Γ, Γ' are discrete subgroups which give $\mathcal{N} = 1$ theories. After the Penrose limit, it is easy to check that $AdS_5/\Gamma \times S^5/\Gamma'$ and $AdS_5/\Gamma' \times S^5/\Gamma$ become identical.

4.2 Penrose limit of the AdS space-time orbifolds

The AdS space-time orbifolds have been discussed in [16] where it has been claimed that they come from the near horizon limits of brane configurations with pp-waves. These brane configurations can be related to brane configurations probing Taub-NUT space by a series of S and T-dualities.[5]

The dual field theories contain closed timelike curves and we do not have a clear picture of field theory with such a property. Nevertheless, we can take the Penrose limit by boosting along the θ direction in five-sphere and expand the metric (2.5) about $\rho = 0, \alpha = 0$, i.e.

$$\begin{aligned} x^+ &= \frac{1}{2}\left(\frac{t}{k} + \theta\right), \qquad & x^- &= \frac{1}{2}R^2\left(\frac{t}{k} - \theta\right), \\ \rho &= \frac{r}{R}, \qquad & \alpha &= \frac{x}{R}, \qquad R \to \infty \end{aligned} \tag{4.5}$$

and x, r, x^+ and x^- fixed. Taking the limit, after the redefinition of ϕ as $\phi - x^+$, we obtain

$$ds^2 = -4dx^+dx^- - (r^2 + x^2)(dx^+)^2 + dr^2 + dx^2 + x^2 d\Omega_3^2 + r^2 d\Omega_3'^2 \,, \tag{4.6}$$

which is exactly the maximally supersymmetric plane wave without any orbifolding. Note also that for the subsector of the gauge theory dual to this background the conformal symmetry has also been restored.

One should note that unlike the maximally supersymmetric Penrose limit of the sphere orbifold, which for large k leads to a plane wave with a finite x^- compactification radius [7], here the story is different and, even for large k, x^- is non-compact.

As we do not know the field theory dual to the space-time orbifold, we cannot give a precise interpretation on how to extract the conformal invariant subsector out of the field theory with closed timelike curves. The closed time-like curves appear because we identify points on the time axis. From the form of the Penrose limit (4.5) we see that, in the limit, the ends of the time-like curve are sent to infinity so the closed time-like do not remain closed anymore after the limit. It would be interesting to clarify this aspect.

5. Rotating strings

The field theory results of [2] relating the energy and the angular momentum can be mapped in the string theory by using the results of [3, 4]. An alternative method was suggested in [9, 10] (see also [11, 12, 27] for more involved examples). It consists of choosing a particular pointlike string classical solution and expanding the $AdS_5 \times S^5$ action to obtain the 1-loop correction to the energy.

[5] One should note that, in general, taking the near horizon limit of a given brane configuration does not commute with T-dualities.

In this section we shall study the example of classical rotating solution which describes a folded closed string stretched along radial direction of AdS and rotating along the orbifold part of the space and moving along the large circle of S^5. The classical energy is a function of two angular momenta S and J but the problem is simpler because there is no mixing between the two. The one loop corrections to the energy allows us to determine terms in the expansion of the anomalous dimensions in the field theory side. The quadratic approximation gives the relation [2, 3]

$$E - J = \sum_{n=-\infty}^{n=\infty} \sqrt{1 + \frac{\lambda n^2}{J^2}}\, N_n \,, \tag{5.1}$$

and the quartic approximation allows calculating the $O(1/\sqrt{\lambda})$ terms.

5.1 Classical solution

5.1.1 Space orbifold

Let us consider the case where the orbifold is defined only in the space like directions and the background is given by (2.3). We will look at a rotating string solution which is stretched along radial direction and rotates along ϕ and θ with the angular velocity ω and ν respectively. The solution is given by

$$t = \kappa\tau \,, \qquad \phi = \omega\tau \,, \qquad \theta = \nu\tau \,, \qquad \rho = \rho(\sigma) = \rho(\sigma + 2\pi) \,, \qquad \alpha = 0 \,, \tag{5.2}$$

and all the other angular variables set to be constant. The Nambu action

$$I = -\frac{1}{2\pi\alpha'} \int d\sigma^2 \sqrt{-\det(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu)} \,, \tag{5.3}$$

becomes

$$I = -4\frac{R^2}{2\pi} \int_0^{\rho_0} d\rho \sqrt{(\kappa^2 - \nu^2)\cosh^2\rho - \left(\frac{\omega^2}{k^2} - \nu^2\right)\sinh^2\rho} \,, \tag{5.4}$$

where

$$\coth^2\rho_0 = \frac{\omega^2/k^2 - \nu^2}{\kappa^2 - \nu^2} \,. \tag{5.5}$$

One should also note that $\rho(\sigma)$ is subject to the constraint coming from fixing the conformal gauge

$$\left(\frac{d\rho}{d\sigma}\right)^2 = (\kappa^2 - \nu^2)\cosh^2\rho - \left(\frac{\omega^2}{k^2} - \nu^2\right)\sinh^2\rho \,. \tag{5.6}$$

For this action, the energy, spin and R-charge

$$E = -\frac{\partial I}{\partial \kappa} \,, \qquad S = \frac{\partial I}{\partial \omega} \,, \qquad J = \frac{\partial S}{\partial \nu} \,, \tag{5.7}$$

are obtained to be

$$E = \frac{4R^2}{2\pi}\kappa \int_0^{\rho_0} \frac{\cosh^2\rho \, d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2\rho - (\omega^2/k^2 - \nu^2)\sinh^2\rho}} \,,$$

$$S = \frac{4R^2}{2\pi}\frac{\omega}{k^2}\int_0^{\rho_0} \frac{\sinh^2\rho\, d\rho}{\sqrt{(\kappa^2-\nu^2)\cosh^2\rho-(\omega^2/k^2-\nu^2)\sinh^2\rho}}\,,$$
$$J = \frac{4R^2}{2\pi}\nu\int_0^{\rho_0} \frac{d\rho}{\sqrt{(\kappa^2-\nu^2)\cosh^2\rho-(\omega^2/k^2-\nu^2)\sinh^2\rho}}. \tag{5.8}$$

One observes that

$$E = \frac{\kappa}{\nu}J + \frac{\kappa}{\omega}k^2 S\,, \tag{5.9}$$

and eq. (5.6) and the periodicity also imply another condition on the parameters

$$2\pi = \int_0^{2\pi} d\sigma = 4\int_0^{\rho_0} \frac{d\rho}{\sqrt{(\kappa^2-\nu^2)\cosh^2\rho-(\omega^2/k^2-\nu^2)\sinh^2\rho}}\,. \tag{5.10}$$

It is useful to define $\hat{S} = kS$ and $\Omega = \omega/k$ and we recognize that in terms of these new variables the situation is the same as $AdS_5 \times S^5$. Therefore one can read the energy dependence on J and S form the ones of [10]. By setting

$$\coth^2\rho_0 = \frac{(\Omega^2-\nu^2)}{(\kappa^2-\nu^2)} \equiv 1+\eta\,, \qquad \eta > 0\,,$$

one finds:

A. *Short string*: $\eta \gg 1$. In this case the energy dependence on S and J is given by

$$E^2 \approx J^2 + 2R^2\, kS, \qquad \text{for} \quad \nu \ll 1\,,$$
$$E \approx J + kS + \frac{R^4}{2}\frac{kS}{J^2}\,, \qquad \text{for} \quad \nu \gg 1\,. \tag{5.11}$$

B. *Long string*: $\eta \ll 1$. In this case one get

$$E \approx S + \frac{R^2}{\pi}\ln\frac{kS}{R^2} + \frac{\pi J^2}{2R^2\ln\frac{kS}{R^2}}\,, \qquad \text{for} \quad \nu \ll \ln\frac{1}{\eta}\,,$$
$$E \approx S + J + \frac{R^4}{2\pi^2 J}\ln\frac{kS}{J}\,, \qquad \text{for} \quad \nu \gg \ln\frac{1}{\eta}\,. \tag{5.12}$$

5.1.2 Space-time orbifold

The background for space-time orbifold is given by (2.5). Although there is a potential problem with the existence of closed time-like curves, one can formally repeat the previous semi-classical analysis of the strings in the space-time orbifold as well, without entering to any problem with the singular fixed points. We shall look at a rotating string solution which is stretched along radial direction and rotates along γ and θ with the angular velocity ω and ν, respectively and is fixed at $\sigma = \pi/2$. The solution is given by

$$t = \kappa\tau\,, \qquad \phi = \omega\tau\,, \qquad \theta = \nu\tau\,, \qquad \rho = \rho(\sigma) = \rho(\sigma + 2\pi)\,. \tag{5.13}$$

JHEP01(2003)028

For this solution the Nambu action (5.3) reads

$$I = -4\frac{R^2}{2\pi}\int_0^{\rho_0} d\rho\sqrt{\left(\frac{\kappa^2}{k^2}-\nu^2\right)\cosh^2\rho-(\omega^2-\nu^2)\sinh^2\rho}\,, \tag{5.14}$$

where

$$\coth^2\rho_0 = \frac{\omega^2-\nu^2}{\kappa^2/k^2-\nu^2}\,. \tag{5.15}$$

Using this action one can find the energy, spin and R-charge as following

$$\begin{aligned}
E &= \frac{4R^2}{2\pi}\frac{\kappa}{k^2}\int_0^{\rho_0}\frac{\cosh^2\rho\, d\rho}{\sqrt{(\kappa^2/k^2-\nu^2)\cosh^2\rho-(\omega^2-\nu^2)\sinh^2\rho}}\,,\\
S &= \frac{4R^2}{2\pi}\omega\int_0^{\rho_0}\frac{\sinh^2\rho\, d\rho}{\sqrt{(\kappa^2/k^2-\nu^2)\cosh^2\rho-(\omega^2-\nu^2)\sinh^2\rho}}\,,\\
J &= \frac{4R^2}{2\pi}\nu\int_0^{\rho_0}\frac{d\rho}{\sqrt{(\kappa^2/k^2-\nu^2)\cosh^2\rho-(\omega^2-\nu^2)\sinh^2\rho}}\,.
\end{aligned} \tag{5.16}$$

The periodicity also implies

$$2\pi = \int_0^{2\pi} d\sigma = 4\int_0^{\rho_0}\frac{d\rho}{\sqrt{(\kappa^2/k^2-\nu^2)\cosh^2\rho-(\omega^2-\nu^2)\sinh^2\rho}}\,. \tag{5.17}$$

We define $\hat{E} = kE$ and $\hat{\kappa} = \kappa/k$ and we recognize that in terms of these new variables the situation is the same as for $AdS_5\times S^5$. Therefore, by using the results of [10], the energy dependence on J and S is

A. *Short string*: $\eta \gg 1$. In this case the energy dependence on S and J is given by

$$\begin{aligned}
k^2E^2 &\approx J^2 + 2R^2\, S\,, &&\text{for}\quad \nu\ll 1\,,\\
kE &\approx J + S + \frac{R^4}{2}\frac{S}{J^2}\,, &&\text{for}\quad \nu\gg 1\,.
\end{aligned} \tag{5.18}$$

B. *Long string*: $\eta \ll 1$. In this case one get

$$\begin{aligned}
kE &\approx S + \frac{R^2}{\pi}\ln\frac{S}{R^2} + \frac{\pi J^2}{2R^2\ln\frac{S}{R^2}}\,, &&\text{for}\quad \nu\ll \ln\frac{1}{\eta}\,,\\
kE &\approx S + J + \frac{R^4}{2\pi^2 J}\ln\frac{S}{J}\,, &&\text{for}\quad \nu\gg \ln\frac{1}{\eta}\,.
\end{aligned} \tag{5.19}$$

5.2 One-loop correction

By knowing the classical expressions for the energy, one can now proceed to compute the 1-loop correction to the classical energy spectrum for the above classical solutions. We will consider only the case of rotation in S^5 i.e. solutions with $\omega = 0$, $\kappa = k\nu$ and $\rho = 0$. Then we will expand the action up to quadratic order for the following fluctuations:

$$t = k\nu\tau + \frac{\tilde{t}}{R}\,,\qquad \theta = \nu\tau + \frac{\tilde{\theta}}{R}\,,\qquad \rho = \frac{\tilde{\rho}}{R}\,,\qquad \alpha = \frac{\tilde{\alpha}}{R}\,. \tag{5.20}$$

This leads to the following bosonic action for the quadratic fluctuations for the background (2.5)

$$
\begin{aligned}
I_2 = -\frac{1}{4\pi}\int d\sigma^2 \Big\{ & -\frac{1}{k^2}\partial_a\tilde{t}\partial^a\tilde{t} + \partial_a\tilde{\theta}\partial^a\tilde{\theta} + \nu^2(\tilde{\rho}^2+\tilde{\alpha}^2) + \partial_a\tilde{\alpha}\partial^a\tilde{\alpha} + \partial_a\tilde{\rho}\partial^a\tilde{\rho} + \\
& + \tilde{\rho}^2\left[\partial_a\tilde{\delta}\partial^a\tilde{\delta} + \sin^2\delta\ \partial_a\tilde{\gamma}\partial^a\tilde{\gamma} + +\cos^2\delta\ (\partial_a\tilde{\phi}\partial^a\tilde{\phi} + -2\nu\partial_\tau\phi - \nu^2)\right] + \\
& + \tilde{\alpha}^2\ \partial_a\Omega_3\partial^a\Omega_3 \Big\}\,. \qquad (5.21)
\end{aligned}
$$

This is the same action that is found by expanding the string action in the maximally plane wave background. To see this consider the following change of variable

$$
\frac{\tilde{t}}{k} = x^+ + x^-\,, \qquad \tilde{\theta} = x^+ - x^-\,, \qquad (5.22)
$$

then in the quadratic approximation and in the light cone gauge where we we have $x^+ = 2\nu\tau$ one finds

$$
\begin{aligned}
I_2 = -\frac{1}{4\pi}\int d\sigma^2 \Big[& -4\partial_a x^+\partial^a x^- - \frac{1}{4}(\tilde{\rho}^2+\tilde{\alpha}^2)\partial_a x^+\partial^a x^+ + \partial_a\tilde{\alpha}\partial^a\tilde{\alpha} + \partial_a\tilde{\rho}\partial^a\tilde{\rho} + \\
& + \tilde{\rho}^2\ \partial_a\hat{\Omega}_3\partial^a\hat{\Omega}_3 + \tilde{\alpha}^2\ \partial_a\Omega_3\partial^a\Omega_3\Big]\,, \qquad (5.23)
\end{aligned}
$$

where

$$
\partial_a\hat{\Omega}_3\partial^a\hat{\Omega}_3 = \partial_a\tilde{\delta}\partial^a\tilde{\delta} + \sin^2\delta\ \partial_a\tilde{\gamma}\partial^a\tilde{\gamma} + +\cos^2\delta\ \partial_a\tilde{\phi}\partial^a\tilde{\phi}\,, \qquad (5.24)
$$

and we have redefined $\tilde{\phi}$ and $\tilde{\phi} - x^+$. As we see this is exactly the string theory on the maximally SUSY plane wave background, which is also due to the fact that the Penrose limit of the background gives the maximally supersymmetric plane wave. Now we can use the results of string theory on the maximally SUSY plane wave background to write down the corrections to the energy spectrum up to order of $\mathcal{O}(1/R^2)$, i.e.

$$
kE - J = \frac{1}{\nu}\sum_{n=-\infty}^{n=\infty}\sqrt{n^2+\nu^2}\ N_n + \mathcal{O}\left(\frac{1}{R^2}\right), \qquad (5.25)
$$

where N_n is the occupation number for 8 sets of the bosonic oscillators.

5.3 Higher order correction

So far we have seen that in the semi-classical approach one cannot distinguish between the half supersymmetric AdS space and AdS space time orbifold in leading quadratic order and it would be natural to ask whether the higher order corrections could see any difference between the AdS and AdS space-time orbifold. Also, a natural question is about the possible differences between the S^5 and AdS_5 orbifolds.

In this section we will study the next order corrections to the action, which are of order $1/R^2$, by keeping the terms which are proportional to $1/R^2$ in the expansion of the previous section for small fluctuations around the classical solution (5.20). Expanding up to order $1/R^2$, we get:

$$I_4 = -\frac{1}{4\pi R^2}\int d\sigma^2\Big[-\rho^2\ \sin^2\gamma\ \partial_a\tilde{t}\partial^a\tilde{t} - \tilde{\alpha}^2\partial_a\tilde{\theta}\partial^a\tilde{\theta} + \frac{\tilde{\rho}^4}{3}(\nu^2 + \partial_a\hat{\Omega}_3\partial^a\hat{\Omega}_3) - \\ - \frac{\tilde{\alpha}^4}{3}(\nu^2 + \partial_a\Omega_3\partial^a\Omega_3)\Big]. \tag{5.26}$$

As compared to the AdS case [10], the action has an extra factor $\sin^2\gamma$ in front of $\partial_a\tilde{t}\partial^a\tilde{t}$.

Let us now consider the semi-classical analysis for the AdS space orbifold and compare the results to the ones for the S^5 orbifolds. The classical solution is now given by a state with high momentum in S^5 part of the string background and localized around $\rho = 0$. We study the small fluctuation around this classical solution as follows

$$t = \nu\tau + \frac{\tilde{t}}{R}, \qquad \theta = \nu\tau + \frac{\tilde{\theta}}{R}, \qquad \rho = \frac{\tilde{\rho}}{R}, \qquad \alpha = \frac{\tilde{\alpha}}{R}. \tag{5.27}$$

For this fluctuations the bosonic part of the string action in the background (2.3) up to $\frac{1}{R^2}$ is given by

$$\begin{aligned} I_2^{(AdS/\Gamma)} &= -\frac{1}{4\pi}\int d\sigma^2\Big[-\partial_a\tilde{t}\partial^a\tilde{t} + \partial_a\tilde{\theta}\partial^a\tilde{\theta} + \nu^2(\tilde{\rho}^2 + \tilde{\alpha}^2) + \partial_a\tilde{\alpha}\partial^a\tilde{\alpha} + \partial_a\tilde{\rho}\partial^a\tilde{\rho} + \\ &\quad + \tilde{\rho}^2\partial_a\left(\frac{\Omega_3}{\Gamma}\right)\partial^a\left(\frac{\Omega_3}{\Gamma}\right) + \tilde{\alpha}^2\partial_a{\Omega'}_3\partial^a{\Omega'}_3\Big], \\ I_4^{(AdS/\Gamma)} &= -\frac{1}{4\pi R^2}\int d\sigma^2\Big[-\tilde{\rho}^2\partial_a\tilde{t}\partial^a\tilde{t} - \tilde{\alpha}^2\partial_a\tilde{\theta}\partial^a\tilde{\theta} - \frac{\tilde{\alpha}^4}{3}(\nu^2 + \partial_a{\Omega'}_3\partial^a{\Omega'}_3) + \\ &\quad + \frac{\tilde{\rho}^4}{3}\left(\nu^2 + \partial_a\left(\frac{\Omega_3}{\Gamma}\right)\partial^a\left(\frac{\Omega_3}{\Gamma}\right)\right)\Big], \end{aligned} \tag{5.28}$$

where Ω'_3 is the S^3 inside of S^5 and the orbifold acts on the AdS part.

To compare the results with the S^5 orbifold, we also need the results for the latter. The loop corrections in this case is very similar to that of $AdS_5 \times S^5$ studied in [10] (see also [27]) and the corresponding expanded bosonic action is

$$\begin{aligned} I_2^{(S^5/\Gamma)} &= -\frac{1}{4\pi}\int d\sigma^2\Big[-\partial_a\tilde{t}\partial^a\tilde{t} + \partial_a\tilde{\theta}\partial^a\tilde{\theta} + \nu^2(\tilde{\rho}^2 + \tilde{\alpha}^2) + \partial_a\tilde{\alpha}\partial^a\tilde{\alpha} + \partial_a\tilde{\rho}\partial^a\tilde{\rho} + \\ &\quad + \tilde{\rho}^2\partial_a\Omega_3\partial^a\Omega_3 + \tilde{\alpha}^2\partial_a\left(\frac{{\Omega'}_3}{\Gamma}\right)\partial^a\left(\frac{{\Omega'}_3}{\Gamma}\right)\Big], \\ I_4^{(S^5/\Gamma)} &= -\frac{1}{4\pi R^2}\int d\sigma^2\Big[-\tilde{\rho}^2\partial_a\tilde{t}\partial^a\tilde{t} - \tilde{\alpha}^2\partial_a\tilde{\theta}\partial^a\tilde{\theta} + \frac{\tilde{\rho}^4}{3}(\nu^2 + \partial_a\Omega_3\partial^a\Omega_3) - \\ &\quad - \frac{\tilde{\alpha}^4}{3}\left(\nu^2 + \partial_a\left(\frac{{\Omega'}_3}{\Gamma}\right)\partial^a\left(\frac{{\Omega'}_3}{\Gamma}\right)\right)\Big]. \end{aligned} \tag{5.29}$$

As we see the leading quadratic action in both (5.28) and (5.29) is identical. This is, of course, expected because the Penrose limit of both of them lead to the same plane wave. On the other hand in the $1/R^2$ order we observe a difference of sign. This is not a very striking feature of the solution, because even in the $AdS_5 \times S^5$ case studied in [10] the contribution to the quartic action of the angular parts of AdS_5 and S^5 come with opposite signs (see equation (4.15) of [10]). But in our case the different sign feature is more dramatic, as it alters the map between the Penrose limit of AdS_5 orbifold and the Penrose limit of S^5 orbifold, which were identical up to the quadratic correction. This is because for the quartic correction, we start probing beyond the plane wave limit.

6. Discussions

In the present project, we have given arguments for the identification of conformal subsectors of S^5 and AdS_5 orbifolds. As the case of S^5 orbifold has been extensively studies before, we have concentrate on the AdS_5 orbifold and have used arguments based on [13] in order to identify the set of states in the two subsectors.

Constructing the corresponding BMN operators for both AdS_5 and S^5 orbifolds, we explicitly showed that the BMN operators for both cases are identical in form, though different in interpretations in the dual gauge theories. Then, we also studied the equivalence between the Penrose limits of $AdS_5 \times S^5$ orbifolds through semi-classical string theory computations. All these analysis confirms the existence of the Z_2 exchange symmetry between the two SO(4) factors of the plane wave isometry group, in accordance with results of [24].

As we work with AdS_5 orbifolds, it should be interesting to expand supergravity fields in terms of harmonics on AdS_5 (as in [22] for S^5) and to project out the harmonics as in [21]. Besides, the orbifold will imply the existence of twisted sectors living on $S^1 \times S^5$ whose description is similar to [19].

Here we also briefly discussed the Penrose limit of half supersymmetric "time-like" orbifolds and showed that it leads to the maximal supersymmetric plane wave geometry. On the other hand taking the Penrose limit in the dual field theory is equivalent to focusing on a definite subsector of the theory, e.g. the BMN subsector for the case of $\mathcal{N}=4$ $D=4$ SYM. Therefore we expect that a subsector of the theory which has closed time-like curves is equivalent to BMN sector of a conformal theory. Noting that the BMN sector is a non-trivial (interacting) subsector, it will be very interesting to see whether this observation can be used to extract information about the dual field theory living on the boundary of space-time AdS orbifold. We also studied semi-classical description of strings in the AdS space-time orbifolds. As we showed it seems that in this case, unlike the case of usual flat space, we do not have (UV) problems. This may in part be related to the fact that the number of orbifold images of a given particle when the particle moves to the fixed point is finite.

Finally we would like to point out that similar to the discussions of [28] one can consider "parabolic" orbifolds of AdS space [29]. These orbifolds can also be half supersymmetric. Unlike the case of [30], in the AdS case the "shift" can only be a rotational twist. It would be very interesting to study the Penrose limit and BMN duals of the theories on AdS_5/Γ^+ spaces discussed in [29].

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JHEP01(2003)028

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